

6.7

Areas of Triangles and Quadrilaterals

- Goals**
- Find the areas of squares, rectangles, parallelograms, and triangles.
 - Find the areas of trapezoids, kites, and rhombuses.

POSTULATE 22: AREA OF A SQUARE POSTULATE

The area of a square is the square of the length of its side, or $A = \text{side}^2$.

POSTULATE 23: AREA CONGRUENCE POSTULATE

If two polygons are congruent, then they have the same area.

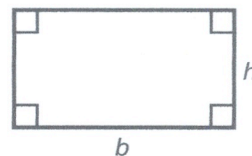
POSTULATE 24: AREA ADDITION POSTULATE

The area of a region is the sum of the areas of its nonoverlapping parts.

THEOREM 6.20: AREA OF A RECTANGLE

The area of a rectangle is the product of its base and height.

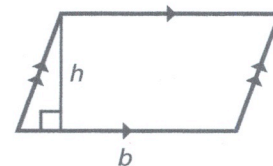
$$A = \text{base} \times \text{height}$$



THEOREM 6.21: AREA OF A PARALLELOGRAM

The area of a parallelogram is the product of a base and its corresponding height.

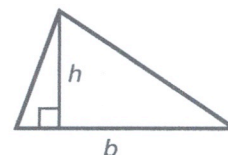
$$A = \text{base} \times \text{height}$$



THEOREM 6.22: AREA OF A TRIANGLE

The area of a triangle is one half the product of a base and its corresponding height.

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

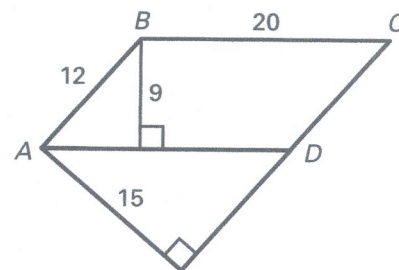


Example 1 *Using the Area Theorems*Find the area of $\square ABCD$.**Solution**Use \overline{AB} as the base. So, $b = \underline{\hspace{1cm}}$ and $h = \underline{\hspace{1cm}}$.

$$\text{Area} = bh$$

$$= \underline{\hspace{1cm}}(\underline{\hspace{1cm}})$$

$$= \underline{\hspace{1cm}} \text{ square units}$$

Notice that you get the same area using \overline{BC} as the base.**Example 2** *Finding the Height of a Triangle*Rewrite the formula for the area of a triangle in terms of h . Then use your formula to find the height of a triangle that has an area of 18 and a base length of 6.**Solution**Rewrite the area formula so h is alone on one side of the equation.

$$A = \underline{\hspace{1cm}} \quad \text{Formula for the area of a triangle}$$

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{Multiply each side by 2.}$$

$$\underline{\hspace{1cm}} = h \quad \text{Divide each side by } b.$$

Substitute $\underline{\hspace{1cm}}$ for A and $\underline{\hspace{1cm}}$ for b to find the height of the triangle.

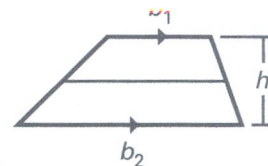
$$h = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Answer The height of the triangle is $\underline{\hspace{1cm}}$.**✓ Checkpoint** Find the area or height of the polygon.

1.	2.	3.
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of the height and the sum of the bases.

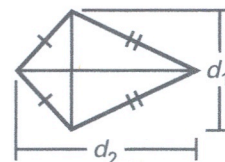
$$A = \underline{\hspace{2cm}}$$



THEOREM 6.24: AREA OF A KITE

The area of a kite is one half the product of the lengths of its diagonals.

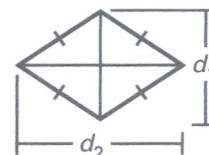
$$A = \underline{\hspace{2cm}}$$



THEOREM 6.25: AREA OF A RHOMBUS

The area of a rhombus is one half the product of the lengths of the diagonals.

$$A = \underline{\hspace{2cm}}$$



Example 3 Finding the Area of a Trapezoid

Find the area of trapezoid JKLM.

Solution

The height of JKLM is

$$h = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}.$$

Find the lengths of the bases.

$$b_1 = JK = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$b_2 = LM = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Substitute $\underline{\hspace{1cm}}$ for h , $\underline{\hspace{1cm}}$ for b_1 , and $\underline{\hspace{1cm}}$ for b_2 to find the area of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

Formula for area of a trapezoid

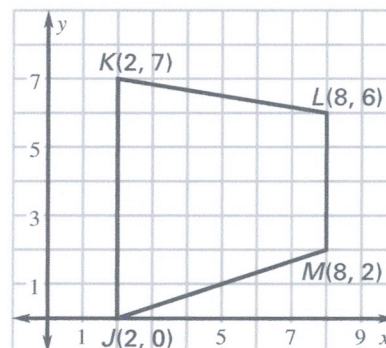
$$= \frac{1}{2}(\underline{\hspace{1cm}})(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$$

Substitute.

$$= \underline{\hspace{2cm}}$$

Simplify.

Answer The area of trapezoid JKLM is $\underline{\hspace{2cm}}$ square units.



Example 4 Finding the Area of a Rhombus

Use the information in the diagram to find the area of rhombus $ABCD$.

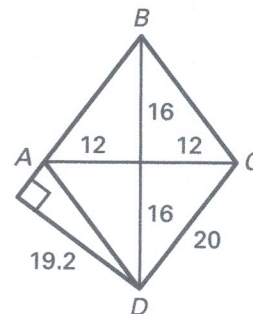
Solution

Method 1 Use the formula for the area of a rhombus. Let $d_1 = BD = \underline{\hspace{2cm}}$ and $d_2 = AC = \underline{\hspace{2cm}}$.

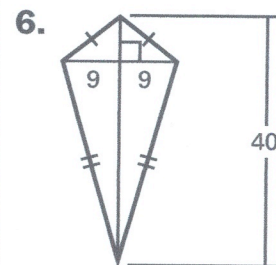
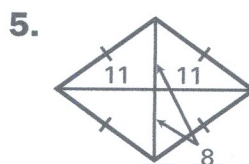
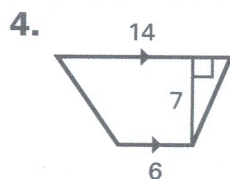
$$\begin{aligned} A &= \underline{\hspace{1cm}} d_1 d_2 \\ &= \underline{\hspace{1cm}} (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) \\ &= \underline{\hspace{1cm}} \text{ square units} \end{aligned}$$

Method 2 Use the formula for the area of a parallelogram. Let $b = \underline{\hspace{2cm}}$ and $h = \underline{\hspace{2cm}}$.

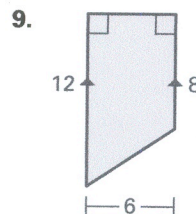
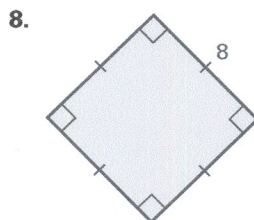
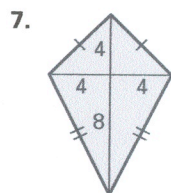
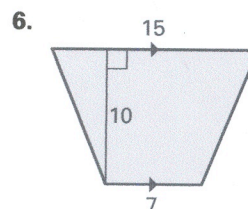
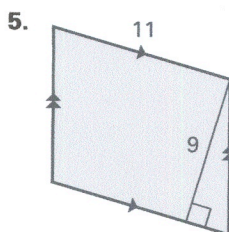
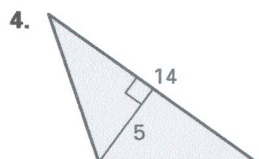
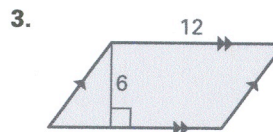
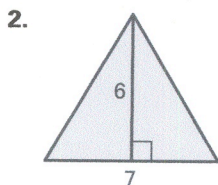
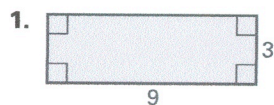
$$\begin{aligned} A &= bh \\ &= (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) \\ &= \underline{\hspace{1cm}} \text{ square units} \end{aligned}$$



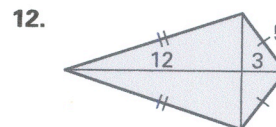
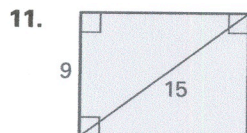
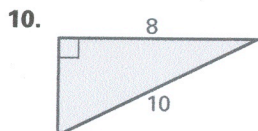
✓ Checkpoint Find the area of the polygon.



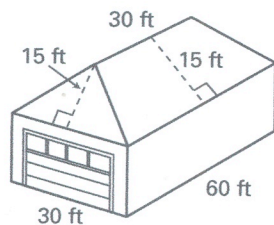
Find the area of the polygon.



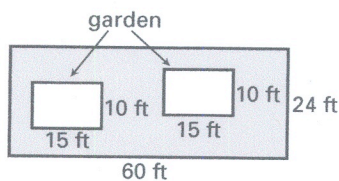
Use the Pythagorean Theorem to find the area of the polygon.



13. The garage roof shown is made from two isosceles trapezoids and two isosceles triangles. Find the area of the entire roof.



14. The Millers have two gardens as shown below. The shaded region represents the lawn that needs to be fertilized. Find the area of the lawn.

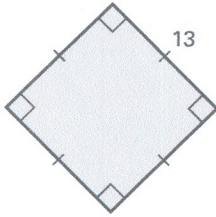


Practice B

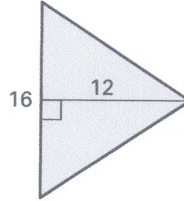
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Find the area of the polygon.

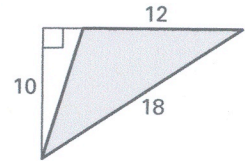
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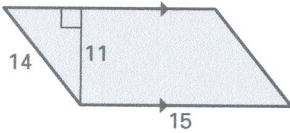
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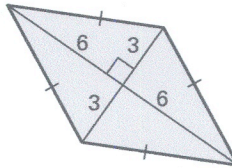
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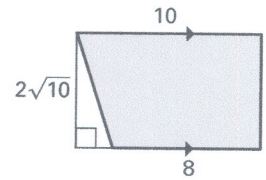
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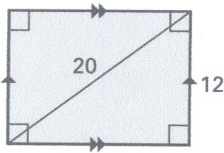
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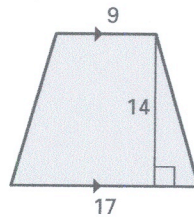
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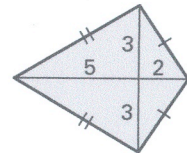
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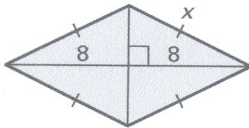


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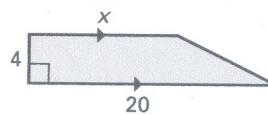


The quadrilateral has an area of 64 square units. Find the value of x .

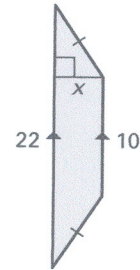
10.



11.

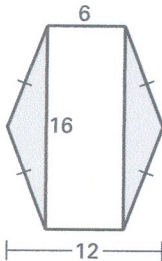


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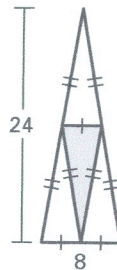


Find the areas of the shaded and unshaded regions.

13.



14.



15.

